MATH 502 HOMEWORK 4

Due Friday, November 15.

Problem 1. *Suppose that* $f : \mathbb{N} \to \mathbb{N}$ *is total recursive. Prove that* $A = \bigcup_n W_{f(n)}$ *is r.e.*

Problem 2. *(a) (Reduction) Suppose that* A, B *are r.e. sets. Prove that there are disjoint r.e. sets* A_0, B_0*, such that* $A_0 \subset A$, $B_0 \subset B$*, and* $A_0 \cup B_0 = A \cup B$

(b) (Separation) Suppose A *and* B *are disjoint* Π^0_1 *sets. Prove that there is a recursive* C*, such that* $A \subset C$ *and* $C \cap B = \emptyset$.

Problem 3. *Prove that the following sets are* Σ^0_3*:*

(1) $Cofin := \{e \mid \mathbb{N} \setminus W_e$ *is finite* $\}$.
(2) $\{(a, b) \mid W_a \subset^* W_b\}$*, where* $A \subset^* B$ *means that* $A \setminus B$ *is finite.*
(3) $\{(a, b) \mid$ *there is a recursive* C *s.t* $A \subset C \wedge B\ C = \emptyset\}$.

Problem 4. *Prove that* $\{e \mid W_e \neq \emptyset\}$ *is* Σ^0_1 *complete.*

Problem 5. *Classify the following in the arithmetic hierarchy:*

(1) $\{e \mid W_e \subset \{0, 1\}\}$.
(2) $\{e \mid W_e \neq \emptyset \wedge W_e$ *is finite* $\}$.